Dielectric measurement setup using a microwave cavity V2

Corwin Shiu

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The purpose of this document is to explore the use of a microwave cavity in order to measure the dielectric properties of STO. The goal of the measurement is to find the dielectric constant and its lost tangent as a function of both frequency and bias voltage. In order for this measurement to be relevant to us, it must be done at high frequencies ($\sim 150GHz$) and at low temperatures ($< 4K$).

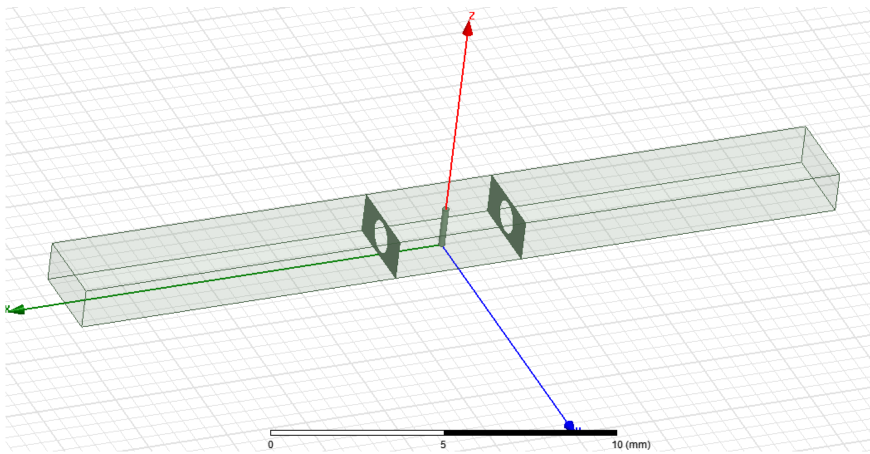

Figure 1: Geometry showing our dielectric test setup. We have a WR10-waveguide (grey) that is aperture coupled to a rectangular cavity (blue). A dielectric (orange) is glued onto one surface of the cavity. We can measure the resonance frequency shift and the change in Q of the first cavity mode with and without the dielectric. These two quantities can be used to find the complex dielectric constant. Additionally we can bias the rectangular cavity to bias the dielectric sample to measure its paraelectric behavior.

1 Rectangular cavity modes

Rectangular waveguide cavity resonators are treated in Pozar in section 6.3. The resonant frequency of an unloaded $TE_{mn\ell}$ or $TM_{mn\ell}$ mode is,

$$f_{mn\ell} = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{\ell\pi}{d}\right)^2} \quad (1)$$

and we have the corresponding fields. For a $\text{TE}_{mn\ell}$ mode we have the following fields with all the prefactors removed (look it up in Pozar 3.2 and don't forget about the 2 or 2j factor)

$$E_z = 0 \tag{2}$$

$$E_x = \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\sin\left(\frac{\ell\pi z}{d}\right) \tag{3}$$

$$E_y = \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin\left(\frac{\ell\pi z}{d}\right) \tag{4}$$

$$H_z = \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin\left(\frac{\ell\pi z}{d}\right) \tag{5}$$

$$H_x = \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{\ell\pi z}{d}\right) \tag{6}$$

$$H_y = \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\cos\left(\frac{\ell\pi z}{d}\right) \tag{7}$$

The VNA at Princeton operates in the WR-10 band ($70-110GHz$). The cleanest measurement of dielectric constant would have the fundamental mode in band. Any cavity perturbation would decrease the resonant frequency, so we want the fundamental mode (TE_{101}) to be on the upper side of the band.

2 Dielectric cavity perturbation

Perturbation of the cavity resonator would cause a shift in the resonant frequency of the cavity. The equation is derived in Pozar 6.7, and the complex form in Sheen's *Amendment of cavity perturbation technique for loss tangent measurement at microwave frequency* in the Journal of Applied physics 102, 014102 (2007).

$$\frac{\omega-\omega_0}{\omega} = \frac{\int_{V_0} dv\ \Delta\mu|\vec{H}_0|^2 + \Delta\epsilon|\vec{E}_0|^2}{\int_{V_0} dv\ \mu|\vec{H}_0|^2 + \epsilon_0|\vec{E}_0|^2} \tag{8}$$

where the integral in the numerator only integrates over the region where the cavity is perturbed. The $\Delta\epsilon$ and $\Delta\mu$ are the dielectric perturbation and in general complex. This form isn't the most useful. We first want to take complex frequency $\omega = 2\pi f b(1 + j/2Q_0)$. If we assume that the material is non-magnetic $\Delta\mu = 0$ and seperate the dielectric component piece into $\epsilon_{STO} = \epsilon' + j\epsilon''$, then,

$$2\left(\frac{f_0-f}{f}\right) = (\epsilon'-\epsilon_0)\frac{\int_{V_s} dv\ |\vec{E}_0|^2}{\int_{V_0} dv\ \epsilon_0|\vec{E}_0|^2} \tag{9}$$

$$\frac{1}{Q_0} - \frac{1}{Q} = \epsilon''\frac{\int_{V_s} dv\ |\vec{E}_0|^2}{\int_{V_0} dv\ \epsilon_0|\vec{E}_0|^2} \tag{10}$$

so by measuring the dielectric shift and the change in quality factor we can estimate both the dielectric constant and the loss tangent

3 DC Biasing

We can bias the dielectric by applying a DC voltage to one of the sides of the cavity and grounding all other sides. If we have a very flat rectangular box as our cavity, then we expect to have parallel plate like fields near the center of the cavity. We can confirm this hypothesis by solving Laplace's equation in a cavity using DC boundary conditions,

$$\nabla^2 V = 0, V(x,y,z=d) = V_0 \tag{11}$$

with $V(x,y,z) = 0$ on all other surfaces. Laplace's equation, $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$ can be solved by separation of variables, $\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} = 0$, which decouple to the equations with the boundary conditions at the surface

$$\frac{d^2X}{dx^2} + \mu X = 0 \qquad X(0) = X(a) = 0 \tag{12}$$

$$\frac{d^2Y}{dy^2} + \lambda Y = 0 \qquad Y(0) = Y(b) = 0 \tag{13}$$

$$\frac{d^2Z}{dz^2} - (\mu + \lambda)Z = 0 \qquad Z(0) = 0 \tag{14}$$

We get solutions,

$$X(x) = \sin\left(\frac{n\pi}{a}x\right) \tag{15}$$

$$Y(y) = \sin\left(\frac{m\pi}{a}y\right) \tag{16}$$

$$Z(z) = \sinh\left(\pi\sqrt{\frac{n^2}{a^2} + \frac{m^2}{b^2}}z\right) \tag{17}$$

so the general solution to Laplace's equation will be,

$$V(x, y, z) = \sum_{mn} c_{mn} \sin\left(\frac{n\pi}{a}x\right) \sin\left(\frac{m\pi}{b}y\right) \sinh\left(\pi\left(\frac{n^2}{a^2} + \frac{m^2}{b^2}\right)^{\frac{1}{2}} z\right) \tag{18}$$

and we can apply the boundary condition at $V(x, y, z = d)$ and use the orthogonality of sines in order to find the coefficients,

$$c_{mn} = \frac{4}{ab \sinh\left(\pi\sqrt{\frac{n^2}{a^2} + \frac{m^2}{b^2}}d\right)} \int_0^b dy \int_0^a dx V_0 \sin\left(\frac{n\pi}{a}x\right) \sin\left(\frac{m\pi}{b}y\right) \tag{19}$$

$$= \frac{4V_0}{\sinh\left(\pi\sqrt{\frac{n^2}{a^2} + \frac{m^2}{b^2}}d\right)} \frac{1 - \cos(n\pi)}{n\pi} \frac{1 - \cos(m\pi)}{m\pi} \tag{20}$$

$$\tag{21}$$

So only odd n, m will have nonzero c_{mn}. We plot the voltage at $z = 0.05mm$ where the middle of the dielectric sample would sit. The figure 2 show that a cavity of dimensions $10mm \times 10mm \times 1.5mm$ would produce uniform electric field in the center of the cavity that is suitable for biasing our STO-sample.

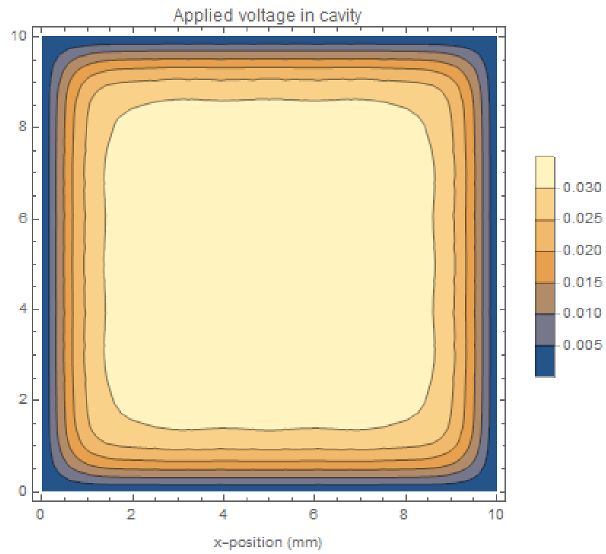

Figure 2: Voltage in the z-plane of a cavity at $z = 0.05mm$. A dielectric sample that is $5 \times 5 \times 0.1mm^3$ glued to the center of the cavity would feel a uniform electric field. The bias to the cavity is $1V$, so we can adjust the fields by increasing or decreasing this bias voltage.

4 Waveguide cavity simulations

4.1 Unloaded cavities

We have to consider two primary factors in designing an optimal waveguide cavity.

(i) The resonant cavity ideally has its fundamental mode in the WR10 waveguide band (70-110GHz). Equation 8 is maximized if we have high E and H fields where our dielectric sample is located. The fundamental mode maximizes power at the center of the resonant cavity - higher order modes will have power spread further away from the center.

(ii) The resonant cavity is excited in predictable ways. While many resonant modes may be allowed for given dimensions, not all modes will be excited by a given aperture placement. For a 2-port cavity operating in the fundamental mode, shown in figure 3, we have very predictable behavior. A fundamental waveguide mode is excited in band. Observing this resonant frequency shift would allow for the measurement outlined above. The difficulty in getting a sample small enough to fit into this cavity ($2.54mm \times 1.27mm \times 2mm$). For a 2-port cavity with the aperture on the side as shown in figure 4, we excite $\text{TE}_{n0\ell}$ modes. If we increase the Z dimensions, we get a whole mess of modes which would be horrendous to disentangle. Note that the transmission amplitudes s_{21} in figure 4 is low. This is because the fundamental mode in the input waveguide will scatter into higher order modes and spread its power over many different modes.

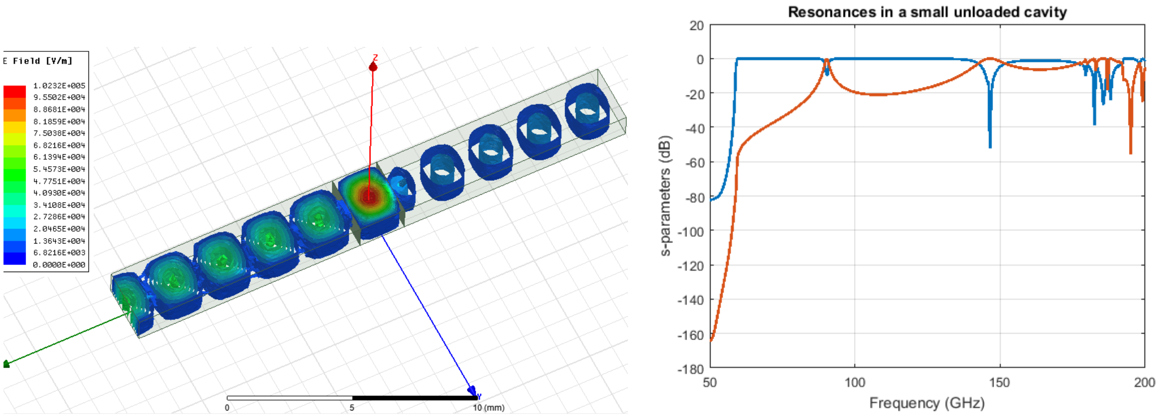

Figure 3: Simulation of a small cavity at the first resonance TE_{101}. The cavity has dimensions ($2.54 \times 1.27 \times 2mm^3$). The small cavity produces a single resonance we can observe in band.

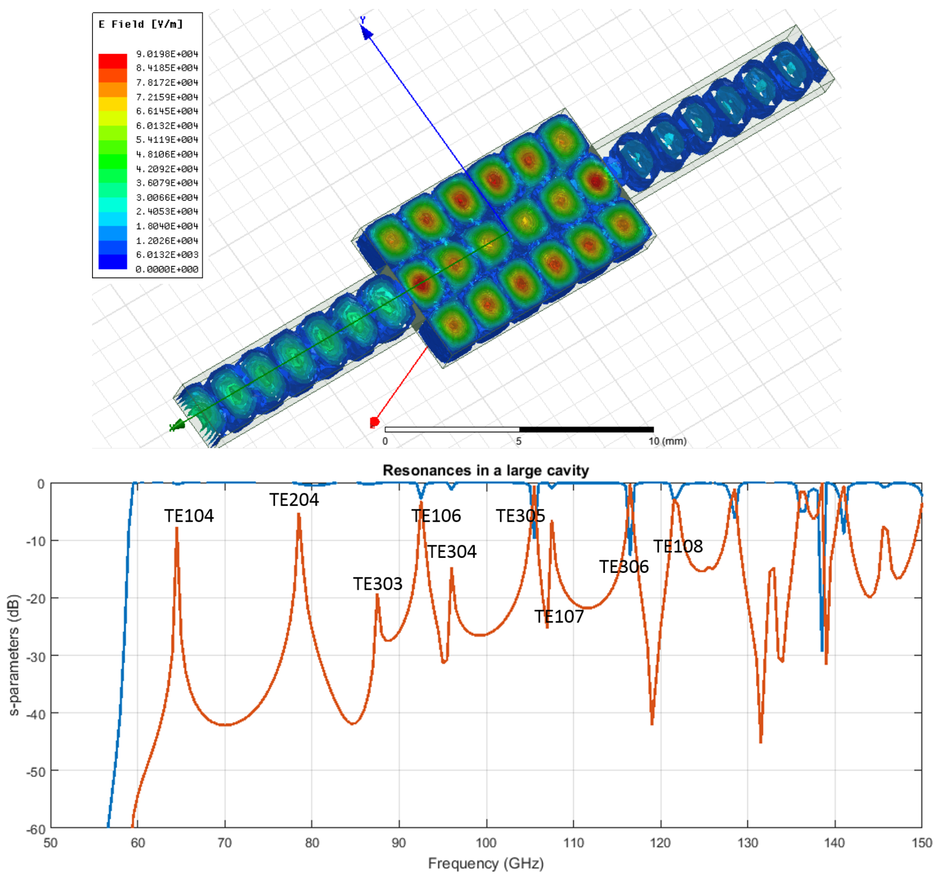

Figure 4: Large cavity that would be easy to slot in a large STO sample, simulated at the TE_{306} mode. We see that we have a huge amount of overlapping resonances, which we can understand in principle understand but would be a pain to disintangle in measurement.

4.2 Loaded cavities

STO can have an extremely high dielectric constant. This might be a problem because the resonance may move out of band. This is shown in figure 5. The resonance shifts to lower frequency, as we expect. However for the large $\epsilon_r = 20$, the fundamental modes drops to below the cutoff frequency of the waveguide - which we would be unable to observe both because its cutoff and out of band. Additionally we see that total transmission s_{21} decreases, but reflection goes up. This is likely due to the impedance of the cavity changing, so we need to tune the aperture to maximize our signal. This is difficult apriori to the measurement.

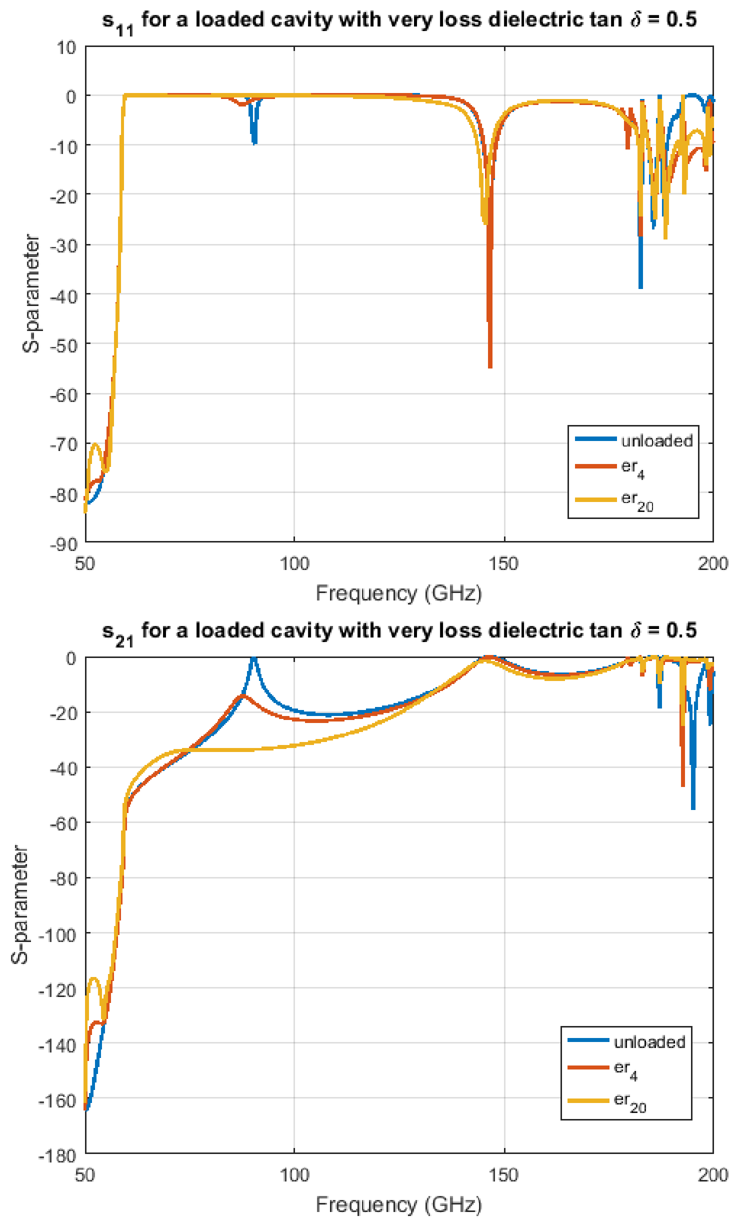

Figure 5: Simulation of a small cavity with a lossy dielectric with $\epsilon_r = 4, 20$ and $\tan\delta = 0.5$. We can see that the fundamental mode shifts with the introduction with the dielectric, and decreases the total transmission. For large ϵ, the resonance shifts to below the cutoff frequency of the waveguide.